Computing solution space properties of combinatorial optimization problems via generic tensor networks (arXiv: 2205.03718)

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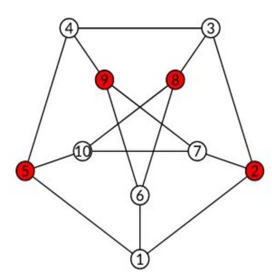
Abstract

We introduce a unified framework to compute the solution space and statistical properties of a broad class of combinatorial optimization problems. The solution space properties include finding one of the optimum solutions, counting the number of solutions of a given size, and enumeration and sampling of solutions of a given size. The statistical properties at finite temperature include sampling configurations, computing marginal probabilities and finding the most likely configuration. Using the independent set problem as an example, we show how all these properties can be computed in the unified approach of generic tensor networks.

Two open source Julia packages: GenericTensorNetworks.jl and TensorInference.jl are developed.

Computational hard problems

Eample: Finding a set of vertices in a graph that no two of them are adjacent.

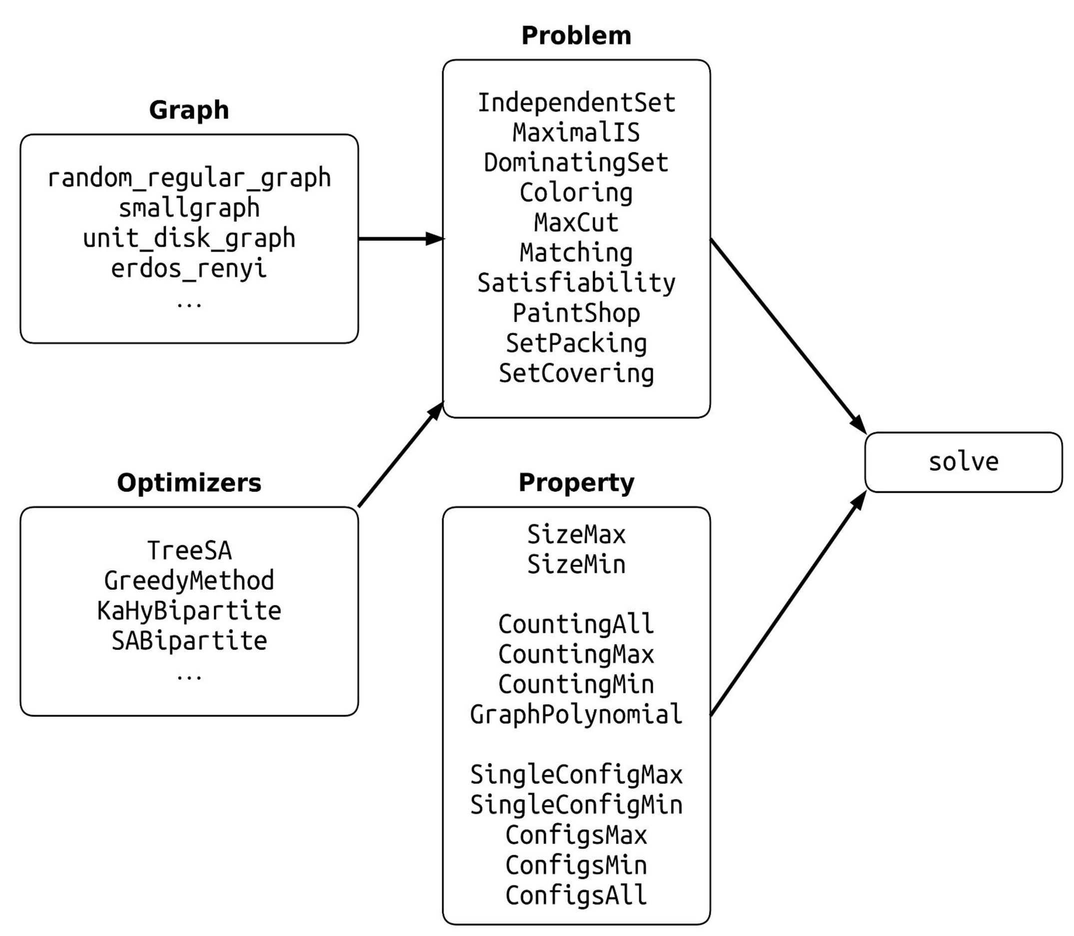


The maximum independent set size of a Petersen graph (above) is 4. It has 5 such equally good solutions.

The solution space properties
- Tier I: Find a best solution
- Tier II: Count the number of solutions of size k
- Tier III: Enumerate/Sample the solutions of size k

The statistical properties at finite temperatures
- Compute the partition function
- Find the (joint) marginal probabilities
- Given an evidence, find the most probable configuration
- Sample from thermal equilibrium

Two packages: GenericTensorNetworks.jl & TensorInference.jl

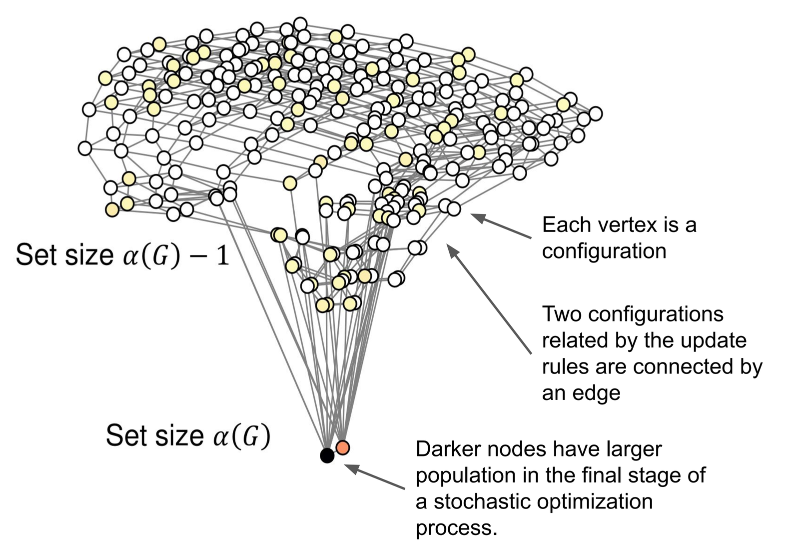


Julia REPL (Terminal)

```
julia> using GenericTensorNetworks, TensorInference, Graphs

julia> graph = Graphs.smallgraph(:petersen)
{10, 15} undirected simple Int64 graph

julia> problem = IndependentSet(graph; optimizer=TreeSA());    # to tensor network with optimized contraction

julia> contraction_complexity(problem)
Time complexity (number of element-wise multiplications) = 2^7.965784284662086
Space complexity (number of elements in the largest intermediate tensor) = 2^4.0
Read-write complexity (number of element-wise read and write) = 2^8.661778097771986

julia> solve(problem, CountingMax(2))[]  # property: counting configurations with maximum 2 sizes
30.0*x^3 + 5.0*x^4

julia> pmodel = TensorNetworkModel(problem, 3.0, mars=[[1], [2, 3], [3, 4]])    # statistical model at β = 3
TensorNetworkModel{Int64, OMEinsum.DynamicNestedEinsum{Int64}, Array{Float64}}
variables: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
contraction time = 2^9.061, space = 2^5.0, read-write = 2^9.656

julia> marginals(pmodel)    # marginal probabilities for variables [1], [2, 3] and [3, 4]
3-element Vector{Array{Float64}}:
 [0.625057241817197, 0.37494275818280304]
 [0.250114483634394 0.37494275818280304; 0.37494275818280304 0.0]
 [0.250114483634394 0.37494275818280304; 0.374942758182803 0.0]

julia> sample(pmodel, 3)    # sample at temperature 1/β
3-element TensorInference.Samples{Int64}:
 [0, 1, 0, 0, 0, 1, 0, 0, 0, 1]
 [0, 1, 0, 1, 0, 1, 0, 0, 0, 1]
 [1, 0, 0, 0, 0, 0, 0, 0, 0, 1]
```

Highlights

(a) It is the method to analyse the quantum algorithm from the configuration space in Science 376, 1209



(b) It can be used to study the overlap gap property

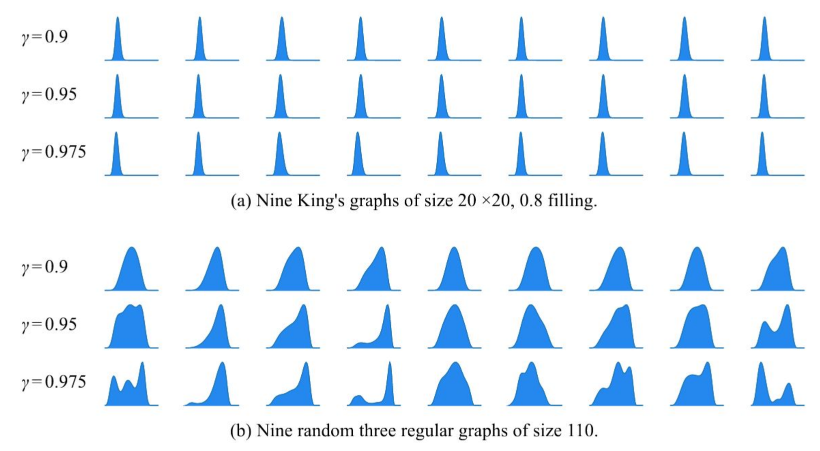


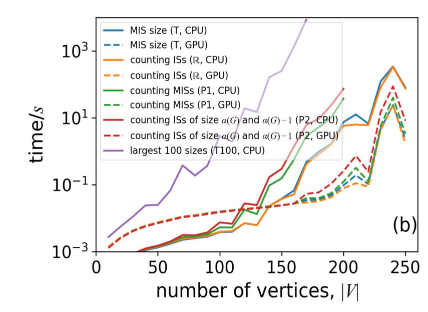
Figure 4: Pairwise Hamming distances distribution for configurations sampled from independent sets with sizes $\geq \lceil \gamma \times \alpha(G) \rceil$. In each plot, the x-axis is the Hamming distance normalized by the total number of vertices and the y-axis is the probability.

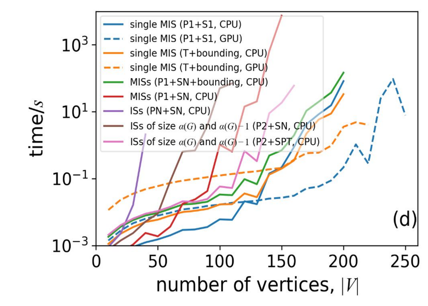
(c) Updated the record of 2D Fibonacci number integer sequence, please refer:
OEIS A006506

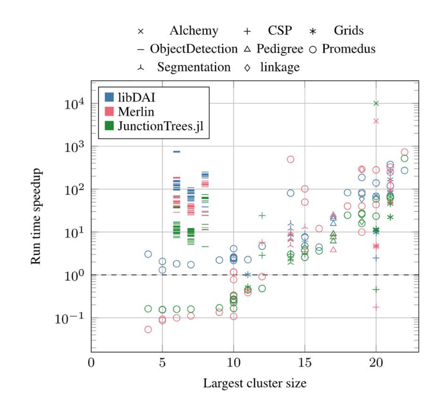
THE ON–LINE ENCYCLOPEDIA OF INTEGER SEQUENCES®
founded in 1964 by N. J. A. Sloane

Why not check out our Github repo?

QuEraComputing/GenericTensorNetworks.jl , where you can find a category of problems solvable by generic tensor networks. Check the list ↗↗↗







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