Atomic Physics for Neutral-Atom Quantum Computing

3 hours · 100 points

Answer all questions. This is a closed-book test; a calculator is allowed. Constants, atomic data and formulas are on the last page. Parts I and II need no justification. In Part II, give two significant figures unless the answer is exact. In Part III, show your reasoning; one significant figure is enough for estimates. Throughout, Ω, Δ, Γ, δ and ω are angular frequencies; a frequency in Hz is written as, e.g., $\Delta/2\pi$.

(12 points) Part I: True/false questions

Mark each statement True (T) or False (F). Each question is worth 2 points.

1. ( ) A tweezer tuned to the blue of a two-level atom's resonance traps ground-state atoms at its intensity maximum.
2. ( ) In a red-detuned laser beam, the ground and excited states of a two-level atom are both shifted down in energy.
3. ( ) The Doppler temperature T_D depends on the wavelength of the cooling light.
4. ( ) ^{88}Sr can be cooled below its Doppler temperature by polarization-gradient cooling on the 461 nm blue line.
5. ( ) An atom excited to a Rydberg state stays trapped at the focus of an 850 nm tweezer, like a ground-state atom.
6. ( ) Estimating a probability $P_1 \approx 1/2$ to a statistical uncertainty of 0.01 takes about 2500 repetitions of the experiment.

(18 points) Part II: Fill in the blanks

Each question is worth 3 points.

7. An atom moves at 1 m/s toward a 461 nm laser beam. In the atom's frame, the laser frequency is shifted by _____ MHz (give the sign).
8. The Doppler temperature of the ^{88}Sr blue line is $T_D =$ _____ mK.
9. A two-level atom on the Rb D2 line is driven at saturation parameter $s = 3$. As the laser frequency is scanned, the scattering rate has a full width at half maximum of _____ MHz.
10. The binding energy of the Rb $70S$ state is $|E_{70}|\ /h =$ _____ GHz.
11. At a Förster resonance, two dipole-coupled pair states of Rydberg atoms are degenerate. The interaction then scales with distance as $1/R^k$ with $k =$ _____.
12. An atom emits photons isotropically. An imaging lens with numerical aperture NA = 0.5 collects a fraction _____ of them.

(70 points) Part III: Problems. Show all your work and justify your answers.

13. (14 points) Atom–light interaction. An atom with levels $|g\rangle$, $|e\rangle$ is driven by a laser with Rabi frequency Ω and detuning $\Delta = \omega_L - \omega_0$. In the rotating frame, $H = -\hbar\Delta\ |e\rangle\langle e| + (\hbar\Omega/2)(|e\rangle\langle g| + |g\rangle\langle e|)$.

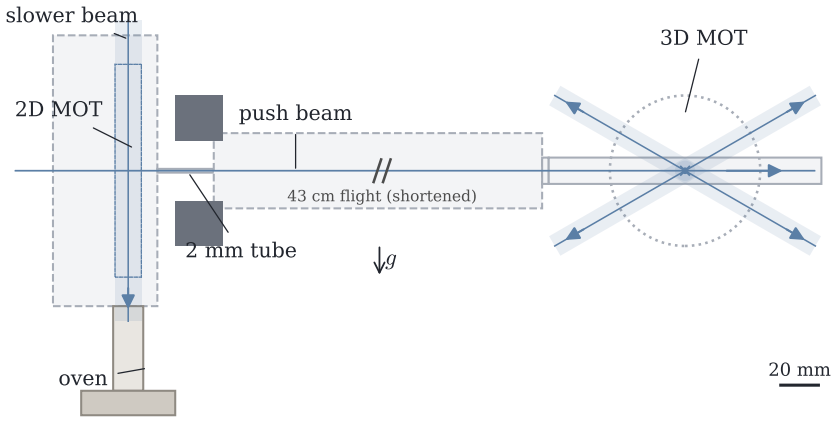
(a) (4 pts) Starting from $|g\rangle$, show that $P_e(t) = (\Omega^2/\Omega'^2)\sin^2(\Omega' t/2)$ with $\Omega' = \sqrt{\Omega^2 + \Delta^2}$. A π pulse ($t = \pi/\Omega$) is calibrated on resonance. Show that the transfer error is second order both in a small relative error ε of Ω and in a small detuning Δ/Ω. Why second order?

(b) (4 pts) Far from resonance ($|\Delta| \gg \Omega, \Gamma$), the ground state has light shift $U = \hbar\Omega^2/4\Delta$. Use the scattering rate on the last page to show that $R_{\rm sc} \approx \Gamma\Omega^2/4\Delta^2$ and $\hbar R_{\rm sc}/U_0 = \Gamma/\ |\Delta|$, where $U_0 = |U|$ is the trap depth. Estimate $R_{\rm sc}$ for ^{87}Rb in an 850 nm tweezer with $U_0/h = 20.8$ MHz. Treat the atom as two-level on the D2 line.

(c) (3 pts) The tweezer is a Gaussian beam with waist w_0, so $U(r,z) = -U_0 I(r,z)/I_0$. Expand to second order in r and z to find the radial and axial trap frequencies ω_r and ω_z. Show that $\omega_z/\omega_r = \lambda/\left(\sqrt{2}\pi w_0\right)$ and evaluate it for $w_0 = 1\mu$m, $\lambda = 850$ nm. Why does a hot atom oscillate at lower frequencies?

(d) (3 pts) The qubit states $|0\rangle$, $|1\rangle$ are hyperfine levels split by $\omega_{\rm hf}$. They are detuned from the excited state by Δ and $\Delta + \omega_{\rm hf}$; assume both couple with the same Ω. Show that their light shifts differ by $\hbar\delta_0 \approx U_0\omega_{\rm hf}/\ |\Delta|$. Estimate $\delta_0/2\pi$ for the trap of part (b). Why does δ_0 matter for the qubit?

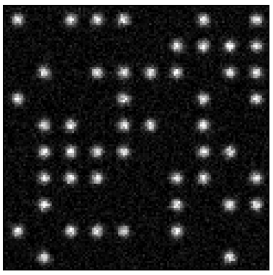
14. (14 points) Laser cooling and trapping. The figure shows a typical ^{88}Sr cold-atom source. Atoms rise from the oven and are slowed by the downward slower beam before they reach the 2D MOT, 7 cm above the oven. The push beam then sends them through a 2 mm tube and over a 43 cm flight into the 3D MOT. The flight is drawn shortened.



(a) (4 pts) Two counter-propagating beams, each with detuning $\Delta < 0$ and saturation $s \ll 1$, act on an atom of velocity v. Write the total force as the sum of two scattering forces, each Lorentzian in the Doppler-shifted detuning. Show that it is linear in v near $v = 0$ and estimate the capture velocity. What do you gain and lose by making $|\Delta|$ larger?

(b) (5 pts) The largest radiation-pressure deceleration is $a_{\max} = \hbar k\Gamma/2m$. Estimate it for the Sr blue line, in units of g. Find the minimum stopping distance of a 300 m/s atom and compare it with the 7 cm. What must be added so that the atom stays resonant while it slows? The push beam launches atoms horizontally at 20 m/s. How far do they fall during the 43 cm flight? Why does gravity matter there but not during slowing?

(c) (5 pts) The image shows one loading attempt of a 10×10 tweezer array from the 3D MOT. Estimate the filling fraction and explain its value. The atoms are then rearranged into N target sites. This needs about $N/2$ moves, each failing with probability ε. Background-gas collisions also remove each atom at rate $1/\tau_{\text{vac}}$ during the sequence time t. Write the probability P of a defect-free array. For $P \gtrsim 1/e$ with $t = 0.1$ s, what do ε and τ_{vac} need as N grows to 10^4? Name one strategy that relaxes these requirements.



15. (14 points) Rydberg atoms. ^{87}Rb is excited from $5S_{1/2}$ to an nS Rydberg state through the intermediate state $6P_{3/2}$, which decays at rate $\Gamma_e = 1/\tau_{6P}$. The lasers at 420 nm (Rabi frequency Ω_1) and 1013 nm (Ω_2) are detuned by Δ from $6P_{3/2}$.

(a) (4 pts) Give the scaling with n^* of the binding energy, the orbit radius, the dipole moment between neighbouring Rydberg states, and the radiative lifetime, with a one-line reason for each. Use second-order perturbation theory to show that $C_6 \propto (n^*)^{11}$.

(b) (4 pts) The effective Rabi frequency is $\Omega_{\text{eff}} = \Omega_1\Omega_2/2\Delta$, and the mean $6P$ population is $(\Omega_1^2 + \Omega_2^2)/8\Delta^2$. Find the probability of scattering a photon from $6P$ during a π pulse, as a function of Δ and Ω_1/Ω_2. Which ratio is best? What prevents making $|\Delta|$ arbitrarily large?

(c) (3 pts) Higher n gives stronger interactions but more sensitivity to stray electric fields (polarisability $\propto (n^*)^7$). At room temperature, blackbody radiation also drives transitions to neighbouring Rydberg states. By what factor does the stray-field Stark shift grow from $n = 70$ to $n = 100$? Why does the room-temperature lifetime grow more slowly with n than the radiative lifetime? Why do experiments mostly use $n \approx 50$–80?

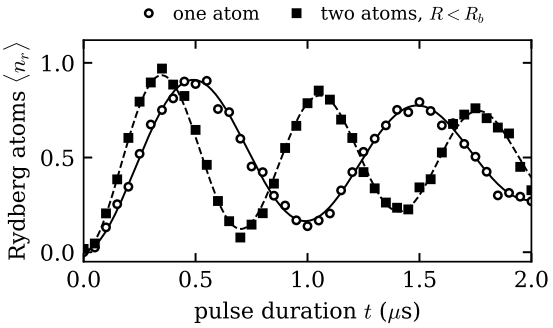
(d) (3 pts) The blockade radius is defined by $C_6/R_b^6 = \hbar\Omega_{\text{eff}}$. Evaluate it for $70S$ with $\Omega_{\text{eff}}/2\pi = 2$ MHz. By what factor must Ω_{eff} change to halve R_b, and, at fixed Δ, how much more laser power does that take? What does this imply for how R_b is set in practice?

16. (14 points) Rydberg blockade and entangling gates. Two atoms are each driven resonantly $|1\rangle \leftrightarrow |r\rangle$ with Rabi frequency Ω; the state $|0\rangle$ is not coupled. When both atoms are in $|r\rangle$ the energy is raised by $V = C_6/R^6$.

(a) (5 pts) For input $|11\rangle$, write H in the basis $\{|11\rangle, |W\rangle, |rr\rangle\}$ with $|W\rangle = (|1r\rangle + |r1\rangle)/\sqrt{2}$. Show that $|11\rangle \leftrightarrow |W\rangle$ couples with matrix element $\hbar\sqrt{2}\Omega/2$, and that the antisymmetric state $(|1r\rangle - |r1\rangle)/\sqrt{2}$ is not coupled. For $V \gg \hbar\Omega$, at what frequency does the pair oscillate? Check your answer against the data in the figure below. Estimate the largest population of $|rr\rangle$.

(b) (3 pts) **Jaksch protocol**: a π pulse on the control atom, a 2π pulse on the target, and a second π pulse on the control with the same laser phase. Assume $V \gg \hbar\Omega$. A resonant 2π rotation returns a state with a sign -1. Find the sign (± 1) acquired by $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$ (control first). Show that the gate equals the controlled-Z gate, $\mathrm{CZ} = \mathrm{diag}(1, 1, 1, -1)$, up to single-qubit $Z = \mathrm{diag}(1, -1)$ gates. Where exactly is the blockade used?

(c) (6 pts) **Error budget.** Rydberg decay gives an error $\varepsilon_d = A/(\Omega\tau)$, and imperfect blockade gives $\varepsilon_b = B\hbar^2\Omega^2/V^2$, where A, B are constants of order one and τ is the Rydberg lifetime. Explain the physical origin of each scaling. Find the optimal Ω and show that the minimum error scales as $(\hbar/V\tau)^{2/3}$. Evaluate it for $A = B = 1$, $\tau = 150\mu$s, $V/h = 100$ MHz. Which helps more: doubling V or doubling τ? By what factor is the error multiplied?

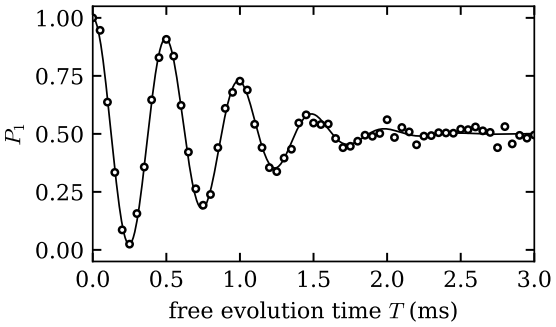


17. (14 points) Coherence and measurement. The qubit is stored in two hyperfine ground states of ^{87}Rb, $|0\rangle$ and $|1\rangle$, in the tweezer of Problem 13. A Ramsey sequence ($\pi/2$ pulse, wait T, $\pi/2$ pulse) with detuning δ gives $P_1 = (1 + \cos \delta T)/2$.

(a) (6 pts) **Thermal dephasing.** Atoms at temperature T_a sample different trap intensities, so their qubit frequencies differ. Model δ as Gaussian with mean $\bar{\delta}$ and standard deviation $\sigma \approx \delta_0 k_B T_a / U_0$ (δ_0 from Problem 13(d)). Show that $\langle P_1 \rangle = \left[1 + C(T) \cos \bar{\delta} T\right]/2$ with contrast $C(T) = e^{-\sigma^2 T^2/2}$. Evaluate T_2^*, defined by $C = 1/e$, for $\delta_0/2\pi = 5$ kHz, $U_0/k_B = 1$ mK, $T_a = 20\mu$K. The figure below shows a Ramsey signal from a hotter sample in the same trap. Read off T_2^* and $\bar{\delta}/2\pi$, and estimate the sample temperature. If the trap is lowered adiabatically to $U_0/4$ before the sequence, does T_2^* get longer or shorter, and by what factor?

(b) (4 pts) **Spin echo.** A π pulse at $T/2$ reverses the sign of the phase accumulated afterwards. For frequency noise $\delta(t)$ with correlation time τ_c, explain when the echo removes it and when it does not. In a tweezer array, the echo extends the coherence from milliseconds to hundreds of milliseconds. What does this tell you about the dominant noise? How would you suppress faster noise?

(c) (4 pts) **Readout.** Each scattered 780 nm photon heats the atom by about $2E_r$. Find E_r/k_B, and the number of photons that would heat an uncooled atom out of a 1 mK trap. Detecting 100 photons with 2% total efficiency needs how many scattered photons? Is cooling during imaging required? Error correction images some atoms in the middle of a computation while their neighbours must keep their state. Why can the imaging light harm a neighbour? Give one remedy.



Constants, atomic data and formulas

$h = 6.63 \times 10^{-34}$ J s, $\hbar = 1.055 \times 10^{-34}$ J s, $c = 3.00 \times 10^{8}$ m/s, $k_B = 1.38 \times 10^{-23}$ J/K, $g = 9.81\text{m/s}^2$, $k_B \cdot 1\text{ mK} = h \cdot 20.8$ MHz.

^{87}**Rb**, $m = 1.44 \times 10^{-25}$ kg	
D2 line $5S_{1/2} \to 5P_{3/2}$	$\lambda = 780$ nm, $\Gamma/2\pi = 6.1$ MHz
$6P_{3/2}$	lifetime $\tau_{6P} = 120$ ns
Qubit	hyperfine splitting $\omega_{\text{hf}}/2\pi = 6.8$ GHz
nS series	quantum defect $\delta_S = 3.13$
$70S$	$C_6/h = 860$ GHz μm^6; lifetime at 300 K: 150μs
^{88}**Sr**, $m = 1.46 \times 10^{-25}$ kg	
Blue line $^1S_0 \to {}^1P_1$	$\lambda = 461$ nm, $\Gamma/2\pi = 30.2$ MHz

- Scattering rate: $R_{\text{sc}} = (\Gamma/2) \cdot s/(1 + s + 4\Delta^2/\Gamma^2)$, $s = I/I_{\text{sat}} = 2\Omega^2/\Gamma^2$.
- Gaussian beam: $I(r,z) = I_0[w_0/w(z)]^2 e^{-2r^2/w(z)^2}$, $w(z) = w_0\sqrt{1 + z^2/z_R^2}$, $z_R = \pi w_0^2/\lambda$.
- Rydberg energies: $E_n = -hR_\infty c/(n^*)^2$, $n^* = n - \delta_\ell$, $R_\infty c = 3.29 \times 10^{15}$ Hz.
- Recoil: $v_r = \hbar k/m$, $E_r = \hbar^2 k^2/2m$.